

MAS212 Linear Algebra I 2007-2008

Coursework 2

Please hand in your solutions of the **starred** exercises by **1pm on Friday 12 October** using the [blue Linear Algebra I Collection Box](#) on the Ground Floor.

Don't forget to put your **name** and **student number** on your solutions, and to **staple** them.

Exercise 1. Give a geometric explanation of why a homogeneous linear system consisting of two equations in three unknowns must have infinitely many solutions.

Exercise* 2. (6 marks)

(a) Consider the matrices A and B given by

$$A = \begin{pmatrix} 1 & -2 \\ -4 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 3 \\ -1 & 5 \end{pmatrix}.$$

Find (i) $A + B$, (ii) $2A - 3B$.

(b) For the following matrices decide which products of two of them exist (including the possibility that a matrix is multiplied by itself) and calculate these products.

$$A = \begin{pmatrix} -2 & -1 \\ 1 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 \\ 1 & -1 \\ 0 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 1 \\ -3 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}.$$

Exercise* 3. (3 marks) Let A and B be square matrices of the same size. Show that

$$(A + B)(A - B) = A^2 - B^2$$

if and only if A and B commute.

Exercise 4. Show that the sum and product of two $n \times n$ upper triangular matrices is upper triangular.

Exercise* 5. (4 marks) Let A and D be square $n \times n$ matrices. Show that if D is diagonal with diagonal entries $\alpha_1, \dots, \alpha_n$, that is $d_{ii} = \alpha_i$ where $D = (d_{ij})$, then

(a) AD is the matrix whose j -th column is α_j times the j -th column of A ;

(b) DA is the matrix whose i -th row is α_i times the i -th row of A .

Exercise* 6. (4 marks) Let A and B be matrices of the same size. Show the following:

(a) $(\alpha A)^T = \alpha A^T$;

(b) $(A + B)^T = A^T + B^T$.

Exercise* 7. (3 marks) Show that if A is invertible then A^T is invertible and

$$(A^T)^{-1} = (A^{-1})^T.$$

[Hint: $(AB)^T = B^T A^T$.]

A cash prize is offered for the best solution received to the following question. Do not hand it in with your coursework, but hand it to me in person either at the Friday lecture or before.

Prize question. A matrix A is said to be **nilpotent** if $A^n = O$ for some integer n . If A is nilpotent and $A^p = O$ but $A^{p-1} \neq O$ for a positive integer p , then A is said to be **nilpotent of order p** .

Construct examples of nilpotent matrices of order p and show that if A is nilpotent then $I - A$ is invertible.